Allocation of Shelf Space: A Case Study of Refrigerated Juice Products in Grocery Stores

Mark G. Brown
Jonq-Ying Lee

The results of this study show how shelf space can be allocated between products in a profit maximizing framework. Cross sectional data were analyzed to determine whether orange juice might have less than optimal shelf space. Estimates of product demands indicate that cross-facings-per-store effects were insignificant; and, hence, only own-facings-per-store effects were used in the analysis. Results indicate that orange juice's actual share of department facings of 51% is less than its optimal share of department facings that range from 80 to 61.6% based on alternative markup assumptions. ©1996 John Wiley & Sons, Inc.

The amount of shelf space allocated to a product in a store has an opportunity cost, knowledge of which allows determination of the most profitable allocation of space between alternative products competing for the store's limited shelf space.[1,2]

Requests for reprints should be sent to Mark Brown, Department of Citrus, Economic Research Department, 2129 McCarty Hall, PO Box 110240, University of Florida, Gainesville, FL 32611-0240.

Factors to consider in determining the optimal allocation include the per unit profit of each product and the demand levels for the products that in turn can be expected to be dependent on such factors as prices, consumer income, and preferences. Preferences, in turn, may be related to such factors as advertising, demographic variables, and shelf space. Shelf space by itself might be considered a form of advertising, putting products on top of consumers' minds, and generally suggesting product popularity levels. Shelf space may also affect demand by reducing consumer search costs. Regardless, a product's sales can be expected to be dependent on the amount of shelf space allocated to the product, as well as characteristics of the space such as location.[3–8] In this case, the optimal allocation of shelf space would depend, in part, on the marginal impacts of shelf space on product demands. Other factors such as the cost of product procurement, storage, insurance, spoilage, and out-of-stock costs may also be relevant, in which case they should be included in the allocation problem.

In this article we examine a line of grocery store

• *Mark G. Brown and Jonq-Ying Lee are Research Economists with the Florida Department of Citrus and Adjunct Associate Professor and Adjunct Professor, respectively, with the Food and Resource Economics Department of the University of Florida.*

Agribusiness, Vol. 12, No. 2, 113–121 (1996)

CCC 0742-4477/96/020113-09

products with similar cost structures. We assume these costs will not change due to a reallocation of shelf space. Hence, we focus on demand and its relationship with shelf space. The article proceeds by examining more closely the question of optimal shelf space using a simple mathematical model. Then an empirical study of refrigerated juice and juice-drink product shelf space illustrates how the model works. The results indicate where store management might look in allocating, and perhaps experiment with, shelf space to maximize profits.

Model

A profit maximization model is used to reveal the basic aspects of the shelf space allocation problem. The problem is to allocate a limited amount of shelf space between n products such that store or store department profits are maximized. Let S be the total fixed shelf space; s_i shelf space allocated to product i; p_i retail price of product i; w_i wholesale price of product i; q_i quantity demanded of product i; and x a vector containing the n retail prices, consumer income, advertising, and other demand explanatory variables. The maximization problem can then be written

$$\max V = m'q, \quad \text{subject to} \quad S = \mathbf{1}'s, \qquad (1)$$

where $m' = (p_1 - w_1, \ldots, p_n - w_n)$, $q' = (q_1, \ldots, q_n) =$, $\mathbf{1}$ is a $n \times 1$ unit vector, and s' $(s_1, \ldots, s_n)$. The term m_i measures the profit per unit of product i, with retail and wholesale prices assumed to be fixed by competitive forces. The demands for products can further be written as functions of x and s, i.e., $q = f(x, s)$. Additional constraints such as lower and upper bounds and/or nonnegativity on s_i might also be included in specifying problem (1).[8]

The Lagrangian for (1) can be written as $L = m'q + r(S - \mathbf{1}'s)$, where r is the Lagrangian multiplier, and the first-order conditions are

$$dL/ds_i = m'dq/ds_i - r = 0, \quad i = 1, \ldots, n. \qquad (2a)$$

$$dL/dr = S - \mathbf{1}'s = 0, \qquad (2b)$$

where, $dq/ds_i = (dq_1/ds_i, \ldots, dq_n/ds_i)'$, with, dy/dz, in general, being the partial derivative of y with respect to z.

In this study, demand is approximated by the double log function $q_i = A_i \prod_j s_j^{e_{ij}}$, where e_{ij} is the elasticity of demand for product i with respect to shelf space for product j, and A_i is a function of x. Equation (2a) can then be written, after multiplying both sides by s_i, as

$$B_i = rs_i, \qquad (3)$$

where $B_i = \Sigma_j m_j q_j e_{ji}$.

Summing (3) over i, the solution for the Lagrangian multiplier can be written as

$$\mathbf{1}'B = r\mathbf{1}'s, \qquad (4a)$$

or

$$r = \mathbf{1}'B/S, \qquad (4b)$$

where $B = (B_1, \ldots, B_n)$ and we have used the constraint $S = \mathbf{1}'s$. One can show that r is marginal profit obtained by relaxing the shelf space constraint by one unit.

Substituting (4b) into first-order conditions (3), the solution for s_i can be written as

$$s_i/S = B_i/\mathbf{1}'B. \qquad (5)$$

Solution (5) indicates that the share of shelf space allocated to product i is product i's share contribution to the profit term $\mathbf{1}'B$. The term B_i can be interpreted as profit resulting from changing product i's allocation of shelf space from zero to s_i, i.e., $B_i = \Sigma_j m_j q_j e_{ji}$, or $B_i = (\Sigma_j m_j dq_j/ds_i)s_i$ and $\mathbf{1}'B$ is the overall profit of shelf space S.

Case 1: Zero Cross-Shelf-Space Elasticities and Constant Elasticity of Substitution

As (1) through (5) show, in general, optimal allocation of shelf space can be expected to be a function of both own-shelf-space elasticities (e_{ii}'s) and cross-shelf-space elasticities (e_{ij}'s). In our empirical analysis, both own- and cross-shelf-space

effects are considered. However, because the cross-shelf-space effects are not statistically different than zero, such cross effects are ignored below to focus on the own-shelf-space effects.

Neither solution (4b) or (5) are in reduced form as the s_i's are on the right sides of the solutions. To obtain an explicit closed-form solution, further structure needs to be given to the problem. For example, suppose all e_{ii} are the same, i.e., $e = e_{ii}$. This is the constant elasticity of substitution (CES) assumption examined in demand and production models.[9] In this case, the shelf space solution can be written as $s_i/S = C_i/1'C$, where $C_i = (m_i A_i)^k$, $k = 1/(1-e)$, and $C = (C_1, \ldots, C_n)$.[a] We assume that the shelf space elasticities are in the zero–one interval $(0 < e < 1)$, assuring that the second-order conditions are met (this seems to be a reasonable assumption given stores have not grossly under- or overallocated shelf space).

Although not in reduced form, (5) can be used to find a solution for s_i, using some iterative procedure (of course, this assumes that the second-order conditions are fulfilled so that a solution exists). For example, initial values for the s_i's can be substituted into the right-hand side of (5) to obtain updated s_i's that can then be substituted into (5) to obtain further updates. This procedure can be repeated until a convergence is achieved, i.e., the values of s_i are the same on both sides of (5).

Case 2: Constant Elasticity of Substitution and Constant Percentage Markups

With some further simplifying assumptions, solution (5) can also be used to examine summary data on shelf space and sales, excluding detailed demand estimates. For example, suppose shelf space for a department in a store is fixed and similarity of products allows the CES approximation to be used. In this case, (5) becomes

$$s_i/S = m_i q_i/m'q. \tag{5a}$$

If all retail prices are marked up approximately by the same percentage g, i.e., $p_i = (1 + g)*w_i$, $g > 0$, then $s_i/S = p_i q_i/p'q$, where $p' = (p_1, \ldots, p_n)$. That is, a product's share of department retail dollar sales equals the share of department shelf space that should be allocated to the product for profit maximization. From this result, we can also see that each product's average dollar sales per unit of shelf space is the same for an optimal allocation, i.e., $p_i q_i/p'q = s_i/S$ or, after rearranging, $p_i q_i/s_i = p'q/S$ for all i.

Case 3: Constant Elasticity of Substitution and Constant Absolute Markups

Similarly, if the CES assumption is maintained and all products have the same absolute markup, $m_i = m$, then

[a] To obtain the CES specification note

$$s_i(e m_i A_i s_i^e/(e m'q))S, \tag{a}$$

or

$$s_i^{(1-e)} = (m_i A_i/m'q)S; \tag{b}$$

or

$$s_i = ((m_i A_i/m'q)S)^{1/(1-e)}, \tag{c}$$

or

$$m_i A_i s_i^e = m_i A_i((m_i A_i/m'q)S)^{e/(1-e)}, \tag{d}$$

or

$$m_i q_i = (m_i A_i)^{1/(1-e)}(m'q)^{-e/(1-e)}S^{e/(1-e)}, \tag{e}$$

or, summing over i

$$m'q = 1'C\,(m'q)^{-e/(1-e)}S^{e/(1-e)}, \tag{f}$$

where as defined previously $C_i = m_i A_i^{1/(1-e)}$ and $C = (C_1, \ldots, C_n)$; hence

$$(m'q)^{1/(1-e)} = 1'CS^{e/(1-e)}, \tag{g}$$

or

$$m'q = (1'C)^{(1-e)}S^e. \tag{h}$$

Substituting (h) into (c) results in

$$s_i = (C_i/1'C)S. \tag{i}$$

$$s_i/S = q_i/\mathbf{1}'q, \qquad (5b)$$

or the share of department quantity sales accounted for by a product equals its share of shelf space. Of course, one would prefer to have knowledge of per unit profits, demand levels, and demand impacts of shelf space so that (5) could be applied more precisely.

Application

Data on shelf space and dollar and gallon sales for juice and juice-drink products in refrigerated departments of grocery stores were examined for consistency with the above theoretical results. The data were provided by Nielsen Marketing Research and are for US grocery stores doing $4 million or greater business annually, for the week ending March 12, 1994. The analysis takes the total shelf space allocated to the juice and juice-drink products as given.[b] Shelf space was measured by number of visible product facings (given juice and juice-drink product similarity, the number of linear inches per facing is expected to be about the same across the products studied).

The product with the largest sales and shelf space in the refrigerated juice and juice-drink department is orange juice (OJ). OJ accounts for more sales and shelf space than all remaining juices and juice drinks (RJ). The question asked here is whether the OJ-RJ shelf space allocation is consistent with our theoretical results. Table I shows the basic data. The shares of department dollar and gallon sales accounted for by OJ are substantially greater than the corresponding shelf space share: OJ had 51% of the department facings versus 72% of the department dollar sales and 68% of the gallon sales. Does this suggest that shelf space is under allocated for OJ? Based on result (5), it is possible that profit per unit of product, demand level, and impact of shelf space on demand could be such that an outcome as in Table I is consistent with profit maximization. However, with similarity of juice and juice-drink products, the CES specification and across-product equality of markups (either in percentage or absolute terms) may be reasonable approximations. In this case, OJ appears to have less shelf space than would be optimal, based on the discussion in the Model section.

To show that OJ has less than optimal shelf space under the constant percentage markup assumption for the CES model, first note that for equal percentage markups across products, profits V are

$$V = k_1(\Sigma_i p_i q_i),$$

where k_1 is some percentage. Given the CES parameter assumptions, the change in profits for a reallocation of shelf space is

$$dV = k_1 e(\Sigma_i \text{ avg}_i \, ds_i),$$

where $\text{avg}_i = p_i q_i / s_i$ or average retail dollar sales per unit of shelf space for product i and ds_i is the change in product i's shelf space. Given the shelf space constraint, note $\Sigma_i \, ds_i = 0$. Substituting the latter result into dV, we find

$$dV = k_1 e(\Sigma_i \text{ avg}_i - \text{avg}_1) ds_i.$$

That is, if product 1 were OJ, profits could be increased by increasing OJ's shelf space and decreasing the shelf space for the remaining products, because as shown in Table I, OJ has the greatest average retail dollar sales per facing of shelf space, i.e., $\text{avg}_i - \text{avg}_1 < 0$ for $i = 2, 3, \ldots, n$, implying ds_2 through ds_n should be negative for an increase in profit.

Demands for OJ and RJ as Functions of Shelf Space

Cross sectional data were used to estimate the demands for OJ and RJ, and the resulting demand relationships were further analyzed to determine whether OJ might have less than optimal shelf space. In sampling US grocery stores, Nielsen has divided the United States into 51 regions. For each of these regions, data on shelf space, dollar

[b] A complete analysis would need to link the amount of shelf space in the refrigerated department to total store shelf space.

Table I. Dollar and Gallon Sales Versus Facings in US Refrigerated Juice and Juice-Drink Department, for Week Ending March 12, 1994

	Orange Juice	Other Juices and Drinks	Department Total
Facing (000)	1,718	1,657	3,375
% Dept. Total Facing	50.9	49.1	100.0
Dollar Sales (000)	34,476	13,530	48006
% Dept Total Dollars	71.8	28.2	100.0
Gallon Sales (000)	9,266	4,318	13,584
% Dept. Total Gallons	68.2	31.8	100.0
Dollars/Facing	20.07	8.17	14.22
Gallons/Facing	5.39	2.61	4.02

Data are for US grocery stores, each with annual sales on all items of at least $4 million.

and gallon sales, consumer income, and population are available. From this data, the logs of per capita gallon sales of OJ and RJ were calculated and used as dependent variables in demand regressions. The explanatory variables for each demand regression were the logs of prices of OJ and RJ; the logs of the average OJ and RJ facings per store; the log of per capita income; an age variable, defined as the percentage of the population over 34 years old; and two regional dummy variables, one for the Northeast and the other for the West.[c] Descriptive statistics for the data are shown in Table II.

Table II. Descriptive Statistics for Regional Data, Week Ending March 12, 1994.

Variable	Mean	Standard Deviation
Weekly OJ Per Capita Gallons	0.036	0.014
Weekly RJ Per Capita Gallons	0.017	0.005
OJ Price ($/gal)	3.704	0.613
RJ Price ($/gal)	3.112	0.374
Annual Per Capita Income (000)	13.804	1.836
OJ Facing Per Store	73.559	12.199
RJ Facing Per Store	67.924	11.017
% Population > 34 Years Old	47.571	3.232
% Sample in		
Northeast	19.608	40.098
West	31.373	46.862
Other regions	49.020	50.488

Data are for US grocery stores, each with annual sales on all items of at least $4 million. Sample is comprised of 51 regional observations.

All explanatory variables were treated as predetermined and independent of the demand equation error terms. If the time interval of an observation were not a week, this assumption might be inappropriate for the price and shelf space variables. However, given such a short time period, prices and shelf space are expected to be essentially fixed by grocery store plans, including promotional specials that typically last for a week or more and provide consumers with some type of deal that does not change over the period offered. In addition, the facings-per-store variables reflect average stocked shelf space; out-of-stocks are assumed to be similar across the regional observations.[d]

Preliminary regression results suggested that the cross-facings-per-store effects are not significant in

[c]The demand specifications can be viewed as approximations resulting from the average (per capita) consumer maximizing utility subject to a budget constraint (average per capita income).

[d]The observed facings per store are stocked facings; out-of-stock facings were not measured. Given similar grocery store management practices and available product supplies across the United States, the percentage of total stocked and out-of-stock facings per store accounted for by stocked facings is expected to be about the same across regions.

explaining OJ and RJ demands. Initially, the demand equations were estimated by ordinary least square (OLS), and t tests were made to test whether the cross-facings effects were zero. For each equation, the cross-facings effect was not different from zero at any reasonable level of significance (the cross-facings' t values were 0.87 and 0.46 for the OJ and RJ equations, respectively). A quasilikelihood ratio test[10] was also used to jointly test the significance of the two cross-facings effects based on seemingly unrelated regression (SUR) estimates. The SUR test result confirmed the OLS result, with the SUR asymptotic chi-square test statistic taking a value of 0.76 with two degrees of freedom.

SUR estimates for the OJ and RJ demand equations, with the cross-facings-per-store effects restricted to zero, are shown in Table III. All parameter estimates are significant at the α = 0.05 level, except that for the Northeast dummy variable in the RJ equation. Because the demand equations are in the double log form, the parameter estimates, except those for the age and regional dummy variables, are elasticities. The own-price elasticities for OJ and RJ are −0.8 and −2.5, respectively, indicating an inelastic OJ demand and an elastic RJ demand. The price of RJ has a negative or complementary effect on OJ demand, while the price of OJ has a positive or substitute effect on RJ demand. The income elasticity for each demand equation is positive, indicating normal type goods. The Northeast and West dummy variable effects are positive and negative, respectively, in each demand equation. The effect of an older age population is positive in both equations. The elasticity of OJ demand with respect to OJ facings per store is 0.48, indicating that a 1.0% increase in OJ facing per store would increase per capita gallon sales of OJ by 0.48%. Similarly, the elasticity of RJ demand with respect to the own-facings variable is 0.55. That is, shelf space for RJ has a somewhat larger own effect than that for OJ. Based on Eq. (5), this result by itself favors RJ in the allocation of shelf space. However, Eq. (5) also shows that markups and demand levels, the latter of which are also dependent on shelf space, are important in determination of optimal shelf space.

Optimal Shelf Space Allocation

The facings-per-store elasticity estimates in Table III were used to solve Eq. (5) for optimal alloca-

Table III. OJ and RJ Demands, SUR Estimates.

	OJ		RJ	
Variable	Estimated Coefficient	*t* Statistic	Estimated Coefficient	*t* Statistic
Constant	−14.956	−5.42	−14.323	−4.98
OJ Facings	0.482	3.73		
RJ Facings			0.555	4.42
OJ Price	−0.779	−2.42	1.137	3.38
RJ Price	−0.821	−2.45	−2.485	−7.14
Income	1.056	3.11	0.856	2.44
Age	0.030	2.81	0.023	2.11
Northeast	0.223	2.59	0.109	1.21
West	−0.314	−3.13	−0.333	−3.23
R^2	0.805		0.661	

Note: The dependent variable is the log of per capita gallon sales; OJ and RJ facings are logs of facings per store for the region; OJ and RJ prices are logs of prices for the region; income is the log of per capita income for the region; age is the percentage of the population over 34 years old for the region; and Northeast and West are regional dummy variables, each equal to one if market is in region, otherwise, equal to zero. Sample is comprised of 51 regional observations.

Table IV. Optimal Percentages of Department Facings for Orange Juice and Other Juices and Juice Drinks, at US Sample Mean

Scenario	Assumed % Markup		Optimal % Department Total Facings		Optimal % Department Total Dollars	
	OJ	RJ	OJ	RJ	OJ	RJ
1	28.0	32.0	79.6	20.4	83.7	16.3
2	26.0	34.0	74.4	25.6	81.4	18.6
3	24.0	36.0	68.4	31.6	78.9	21.1
4	22.0	38.0	61.6	38.4	76.1	23.9
0	—	—	50.9	49.1	71.8	28.2

	Dollars/Facing			Total Dollars (000)		
	OJ	RJ	Department	OJ	RJ	Department
1	15.88	12.09	15.11	42,660	8,333	50,993
2	16.44	10.94	15.03	41,306	9,438	50,745
3	17.17	9.96	14.90	39,667	10,607	50,274
4	18.13	9.13	14.68	37,714	11,819	49,533
0	20.07	8.17	14.22	34,476	13,530	48,006

Note: Scenario 0 is the average for the United States as shown in Table I.

tion of shelf space at the US sample mean.[c] Alternative markups were assumed based on data on OJ and RJ markups in 14 grocery stores in the Midwest. Four scenarios were examined. On average, the markup as a percentage of the retail price was 28% for OJ versus 32% for RJ in the Midwest stores. This set of markups, plus three other sets, with larger markups for RJ and smaller markups for OJ (26% for OJ versus 34% for RJ, 24% for OJ versus 36% for RJ, and 22% for OJ versus 38% for RJ) were examined. The solutions to the latter three sets of markups, which of course favor allocation of shelf space toward RJ, are helpful in determining whether OJ shelf space is underallocated. The levels of OJ and RJ demands, dependent on prices, income, and shelf space, are also important in determination of optimal shelf space. The solution to Eq. (5) brings together the assumed markups, estimates of the facings-per-store elasticities, and estimates of demand levels.

Table IV shows the simulation results. Based on the average markups for the sample of 14 stores, OJ would receive nearly 80% of the refrigerated juice and juice-drink department facings with total department sales in the United States increasing by about $3 million per week or 6.2%. With the estimated facing elasticities being less than unity, average dollars per facing decrease for OJ and increase for RJ and the department, as more shelf space is allocated to OJ and less to RJ.

At the present allocation in Table I, the gain in profit resulting from allocating one more facing to OJ is the OJ markup (0.28) times the OJ dollars per facing (20.07) times the OJ facings elasticity (0.48) or $2.70; the loss in profit resulting from allocating one less facing to RJ is the RJ markup (0.32) times the RJ dollars per facing (8.17) times the RJ facings elasticity (0.56) or $1.46; hence,

[c]An intercept for each demand equation, calculated as the difference between the log of US per capita gallon sales and the product of the facings-per-store elasticity times the log of US mean facings-per-store, was used as the base for the alternative simulations.

the net profit gain is $1.24. These results indicate that, at present, OJ has a much smaller allocation of shelf space than suggested by our simple profit maximization model.

To better understand whether OJ's shelf space is really underallocated, we turn to the alternative markup assumptions favoring RJ. OJ's optimal share of department facings ranges from the 80% based on the average markups to 61.6% based on a markup for RJ of 38% versus a markup for OJ of 22%. That is, even for a relatively high RJ markup compared to that for OJ, our profit maximization model indicates the present allocation of refrigerated shelf space provides OJ with too little space.

There may be various reasons for the present allocation of shelf space such as slotting payments by manufacturers to grocery stores to get their products on shelves, and, perhaps a demand for variety by consumers to which grocery stores may be responding to maintain a base of customers. Further research might consider such possibilities, perhaps, in a profit maximizing framework as suggested here. In addition, one might argue that the aggregation level of the RJ category in the present study biases the results. For this study disaggregated data for RJ were not available. Perhaps, analysis of disaggregated data for the various juice drinks in the RJ category would yield different results. However, given the present information analyzed here, it appears that the average grocery store could increase profits by increasing the amount of shelf space allocated to OJ.

Conclusions

The results of this study show how shelf space can be allocated between products in a profit maximizing framework. The basic problem and equations might also be useful for allocation of other fixed factors among alternative uses. For example, a fixed amount of advertising expenditure might be allocated between different products or different markets to maximize profits or sales;—the question of an optimal allocation can be examined through the advertising impacts on demands and profits using the same approach outlined here.

To fully apply the results of this study, one needs estimates of product demands, including the impacts of shelf space or any other similar factor being examined, along with product markup or per unit profits. Demand interactions between shelf space and factors such as advertising and promotion might also be considered in an application. Given the foregoing information, shelf space allocation can be determined straightforwardly by (5) or some variant with an alternative demand specification, i.e., the double log demand specification in (5) need not be maintained. Using this approach, adjustments of shelf space for demand changes resulting from changes in the explanatory variables under consideration might be considered.

References

1. J. Cairns, "Allocate Space for Maximum Profits," *Journal of Retailing, 39,* 41 (1963).
2. K. Cox, "Effects of Shelf Space Upon Sales of Branded Products," *Journal of Marketing Research, 7,* 55 (1970).
3. H. Pauli and R.W. Hoecker, *Better Utilization of Selling Space in Food Stores: Relation of Size of Shelf Display to Sales of Canned Fruits and Vegetables,* Marketing Research Report No. 30, U.S. Government Printing Office, Washington DC, 1952.
4. J.A. Kotzan and R.V. Evanson, "Responsiveness of Drug Store Sales to Shelf Space Allocations," *Journal of Marketing Research, 6,* 465 (1969).
5. R.E. Frank and W.F. Massy, "Shelf Position and Space Effects on Sales," *Journal of Marketing Research, 7,* 59 (1970).
6. E.E. Anderson, "An Analysis of Retail Display Space: Theory and Methods," *Journal of Business, 52,* 103 (1979).
7. J.B. Wilkerson, J.B. Mason, and C.H. Paksoy, "Assessing the Impact of Short Term Supermarket Strategy

Variables," *Journal of Marketing Research*, *19*, 72 (1982).

8. M. Corstjens and P. Doyle, "A Model for Optimizing Retail Space Allocations," *Management Science*, *27*, 822 (1981).
9. H.R. Varian, *Microeconomic Analysis*, W.W. Norton, New York, 1978.
10. A.R. Gallant and D. Jorgenson, "Statistical Inference for a System of Simultaneous, Non-Linear, Implicit Equations in the Context of Instrumental Variable Estimation," *Journal of Econometrics*, *11*, 275 (1979).